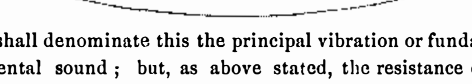
THE MECHANICS AND MATHEMATICS OF MUSICAL VIBRATIONS.

BY SPENCER B. DRIGGS.

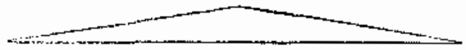
When a string of proper length and thickness is strained to a tension that will cause it to produce 440 double vibrations—that is, to pass the center or resting position 880 times and to make 440 vibrations to the extreme on either side of the center per second—it gives a sound corresponding to pitch *A*. Simultaneously with the commencement of vibration, the string divides itself into aliquot parts, forming nodal points where the vibrations seem to cross. These aliquot parts, by their additional vibrations, produce the octave, the twelfth, fifteenth and nineteenth to the fundamental or pitch sound *A*, and are the *natural* harmonies accompanying every musical sound. And although the normal or pitch sound is obtained *only* by the vibration *of the entire length* of the string between the two bearing points (which would render the secondary or intermediate ones unnecessary so far as relates to the pitch of sound), it is, however, impossible to avoid these harmonies while the string must act against the air with such rapidity and force; and it is quite right that they cannot be avoided as will be hereafter seen. The point of contact being pressed or driven farther out of line than any other portion of the string, will return also further in an opposite direction when free from pressure; and as the air cannot act with equal resistance on all parts, that is, the middle or point of contact cannot move through a greater space in the same length of time that other portions do through a less space, without producing a higher or more acute sound, there appears an absolute necessity for these counter-vibrations, which soften down the otherwise harsh and intolerably unpleasant sounds that must occur in their absence. These nodal points are, however, *only* such to the *intermediate* vibrations, and must of necessity change their position (not relatively) or places with each vibration of the whole string.

I cannot too fully impress the fact that the *whole* string continues to vibrate as long as the *pitch sound* is given, and no longer, and that these vibrations *alone* give the proper pitch sound, which is readily seen by placing an edge upon the string at any of its harmonic nodes, when the principal vibrations at once cease, and the harmonies become the prominent sounds. For example, place the edge at the middle, and the octave sound continues clear and loud. Then place the edge at one-fourth the length, and the second octave or fifteenth becomes prominent and clear, and at the same time the fourth is plainly given by the other three parts. And although these divisions continue to vibrate, nothing can be heard from the whole or *principal* vibrations from the instant the string is touched, which clearly shows that they had ceased when the harmonies became prominent. But notwithstanding the harmonizing influences of these small divisions, any string may be forced to produce exceedingly harsh and unpleasant sounds, by applying a force to put it in vibration beyond its power to instantly recover, or by bringing in contact with it a substance so hard and inelastic that, at the point of concussion, it is forced too suddenly beyond the power of the other portions to sympathize with it, when the result will be harsh and totally unmusical sounds. A very correct idea of this may be obtained by witnessing the various sounds that a fine violinist can produce upon any one string of his instrument, without any change whatever except in the manner of applying and drawing the bow. He can draw the bow so evenly and delicately that only a small portion of the string can be heard to vibrate, producing thereby the most pleasing high sound or one of the harmonies natural to the whole string (and if he wishes to change the harmonic he has to change the length of the string by placing his finger upon it) and, by increasing the power, can cause the whole string to vibrate and produce its regular pitch sound, and so on increasing the power until the friction of the bow will not allow the string to recover its original position at each vibration, when it becomes overpowered and the sound evolved is extremely harsh and "scraping." So also a piano-forte hammer of wood, and without covering, coming in contact with the string will produce some of the most unpleasant and discordant sounds imaginable and so destroy the pitch sound that it can only, with great difficulty and by the most acute ears, be distinguished. This is more perceptible in the lower than in the higher notes. In the other extreme, the hammer may be made so soft, by adding too great a thickness of elastic covering, as to nearly or quite destroy the desired tone, still using the same force of blow as in the former case, which is caused by the fact that the elasticity of the hammer is greater than that of the string, and continues to go forward in the direction sent until *after* the string has reached its furthermost point out of line, and in returning carries the hammer back whose cushion is still against it sufficiently to act as quite a damper to the freedom of vibration. Hence it becomes necessary, to a purely musical evolvement, that the hammer should *leave* the string with as much facility as it *strikes* it. The most sensitive and correctly educated ear, as well as nicety of mechanical skill, are required to graduate and regulate the elasticity of each hammer to that of the particular string it is designed to actuate. For those reasons, all or nearly all additions and appliances in the way of levers and springs for the purpose of relieving the action from the weight of the hammer, or otherwise attempting to facilitate the repeating qualities of piano-forte "actions," have been found by experience to be quite impracticable and often positively injurious, as in fact all such additions are so many impediments to that freedom of action which is absolutely indispensable to the production of pure tone. The common "French action," as it is usually called, I believe to be the best for developing the full capacity and the finest qualities of that queen of musical instruments, and the principle of mechanics upon which it is constructed will admit of its being made as perfect a repeating action as the most fastidious performer could desire. This can be accomplished by enlarging the distance between the center in the flange butt and the point in the hammer butt upon which the fly or "jack" actuates the hammer, to nearly double that which is now commonly used, or say to eleven-sixteenths of an inch, and by moving the balance rail of the keys also, to meet the above change, in such proportion as will retain the proper depth of touch for the keys and the proper drop of hammer, or distance it is to move before reaching the string, and also by checking the hammer, after striking, somewhat nearer the string than is usual. By these simple changes of proportions, which make no addition to the expense of construction, the power and promptness of the action will be greatly increased, and it will also be far more durable, as the strain upon the centers and, in fact, upon the whole action will be greatly relieved.

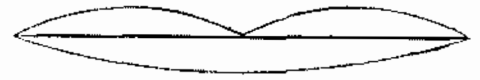
I will now consider and more particularly explain the vibration of a string, as per result of my mathematical calculations and experiments, and believe the following to be not only an original but a perfect solution of one of the most difficult problems that have received the attention of philosophers and mathematicians for ages, and can reasonably say that the simplicity of its truth is the only thing that renders it wonderful that it was not discovered by every one who has heretofore studied or written upon the subject, and more particularly so as it involves the laws that control *all* vibration. When a string of a given length is put in vibration and its tension, thickness, temper and the force applied to start it are all in proportion to the length and cohesive power of its material, it will be found to give, as the most prominent, its proper pitch sound, which is evolved by the vibrations of its *entire* length between the two resting points, thus—



I shall denominate this the principal vibration or fundamental sound; but, as above stated, the resistance of the air will not allow the middle portion to move through a space so disproportioned to that through which the parts nearer the bearings do, in the same length of time, and more particularly to continue in the angular movement or shape in which the point of concussion must necessarily compel it to commence, whether by pressure or percussion, thus—

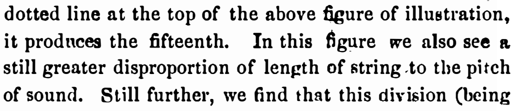


Instantly when freed, an additional vibration is commenced (or when put in vibration by means of a blow or other sudden concussion, these additional or subordinate vibrations commence simultaneously with the principal) whose one node is at the middle of the string and the others at the ends or original bearing points, thus—



These two halves each vibrate twice, by their own additional vibrations, to the whole string once, and each produces the octave to the fundamental note. This division of the vibrations, as well as the others mentioned below, the string is compelled to make in order to preserve its own cohesion and identity. And when these first subordinate vibrations, as shown in the above figure, mingle with the principal, that is to say, when they so unite in motion as to move in the same direction as the principal, as a matter of course during the time that the halves are passing over the whole, either way, their motion is accelerated one-third by being added to the principal, when each octave division produces the musical twelfth, which proves that the velocity with which a string passes through the air regulates the *pitch* of the sound it produces, and not necessarily the *number* of vibrations it produces in a given length of time. But it is also true that a string one-third shorter than either of the above octave divisions (one-sixth the length of the whole string) would of its own principal vibrations produce the said twelfth, and the only way it could pass through the air with sufficient velocity to produce that high sound would be to make one-third or fifty per cent more vibrations in number in the same length of time. Assuming that the whole string first above mentioned is strained to its full vibrating tension, which any string should be to produce the fullest sound of which it is capable, it will be readily seen that so long a string as the one-half of the whole would be, could not recover itself and maintain its identity when compelled to vibrate with sufficient rapidity to produce so high a sound, as the force necessary to start it with that velocity would instantly overpower and break it. And again, when these half parts move in the *opposite* direction to that of the whole string, each half part gives the fundamental note exactly the same as the principal. Then if they are subordinate (these half parts), which they really are, and wholly depend upon the principal for their vibrations, as a matter of course they must go *with* the whole string, and their motion through the air must be accelerated and retarded alternately with each vibration of the principal. And although the *number* of vibrations per second are the same and unchanged in any of their relations, it can be readily seen that the same amount that is added to, to produce the twelfth, when moving *with*, must be taken from when moving *opposite to* the principal, which will result as above stated. The same principle is equally applicable to each division that the string makes for the subordinate vibrations. Whatever is added when they coincide must be deducted when they are opposed in motion in regular alternation. By way of illustration I will suppose a man to run at the rate of 10 miles an hour on the top of a railroad car that is running at the rate of 20 miles per hour. When moving in the same direction as the train he passes through the air at the rate of 30 miles the hour, and when running in the opposite direction his real headway is reduced to 10 miles per hour.

The next division of the vibrations is into fourth parts, the two middle sections forming additional vibrations and producing again the octave to the fundamental note, the same as the first-mentioned division, thus—



When its motion unites with that of the principal it produces the twelfth; and when its movement is united with that of the two first-mentioned octave divisions, and having its nodes upon their centers, as shown by the dotted line at the top of the above figure of illustration, it produces the fifteenth. In this figure we also see a still greater disproportion of length of string to the pitch of sound. Still further, we find that this division (being one-half the length of the whole string) gives, at different times during vibration, four distinct sounds, as follows, viz: 1st, the fundamental, when moving opposite to the principal; 2d, of its own separate vibrations it gives the octave; 3d, when added to the principal it gives the twelfth; and 4th, when added to the first octave division it gives the second octave or fifteenth, as above stated.

When the proportions of thickness, tension, length, &c., as before stated, are balanced, I believe the above *four* divisions to be *all* that any string makes while vibrating, notwithstanding the various contortions and impossible shapes it has been made to assume in the imagination and figurings of some writers on the subject, the proof of which I shall endeavor to give below. There are, however, some additional vibrations from the above five nodal points. For instance, each fourth part has a vibration of its own, which sounds the fifteenth, and when its motion is *with* that of the principal it produces the seventeenth, which frequently becomes the most prominent of the harmonic vanishings, and when the string is too thick or overstrained, the seventeenth not unfrequently becomes unpleasantly prominent with the principal; and so on, still higher, when united with the movement of the first-mentioned octave divisions, but as these are not distinguishable by the the ear, they are less important. There may be also still one other section of separate vibration, viz: taking three of the above four equal divisions; but this is never a prominent sound, except when the finger or other substance is placed upon the string at one-fourth its length, to stop the principal, when this division becomes quite prominent and sounds the fourth. When the string is in full vibration, this section must give the octave; as also the twelfth whenever its own movement is combined respectively with, and added to, one of greater or less velocity than itself, on the same principle as above mentioned.

That a string may be so long and its tension so little or so great in proportion to its length, or when these proportions are right, the manner and power of vibrating it may cause it to form a more compound motion for the instant, I have no doubt, as I have already shown in playing the violin and piano-forte. But as these calculations and experiments have been made for the purpose of analyzing purely musical vibrations, I shall not attempt to go farther with this investigation than its uses extend. I have no doubt, however, that the foregoing will be found to be perfectly true in all the particulars that relate to the laws of vibration; and the few exceptions of certain disproportions which have been mentioned would aid in establishing that as fact. For example: When a string is too *thin* for its length, the lower harmonies—the fourth and the fifth—will become altogether too prominent to admit of proper or pleasing harmony when used in connection with other notes to form a full chord; and, on the other hand, when the string is too *thick* for its length, the higher harmonies stand out with equally unpleasant prominence; by the presence of either of which an acute ear will readily detect the want of proper proportion. Before proceeding with the proofs I will merely state that I have made probably about one hundred different experiments to arrive at a satisfactory result. Many of the experiments were made upon different stringed instruments of music before I found that the sympathetic connection with other strings quite effectually destroyed the possibility of arriving at any definite conclusion. Oftentimes the harmonies from other strings became more prominent than those on the string under experiment. So any one who may wish to prove for himself the truth of what I have written will do well to have but a single string at a time, arranged on a sounding-board by itself, and as free from any and all sympathetic sounds as possible. I found this to be absolutely necessary.

The proofs of the foregoing assertions are as follows:—

First: That the string divides itself into the four parts, as above stated, will be seen by placing the finger at any of the nodal points mentioned, while vibrating, when the pitch of the harmonic sounds described (less the vibrations of the principal, which, of course, cease when the string is touched) will be clearly distinguishable in the absence of all other sounds.

Second: By moving the point of contact away from these nodal points—say to one-third the length of the string, instead of the fourth—the identity of this string is fully lost, and it becomes the same as two other separate strings, of two-thirds and one-third each the length of the original, by each section dividing its vibrations in the center and fourth parts, precisely the same as did the original, which is shown in the fact that the sounds are the same in all their proportions as the original string, except that the two-thirds part has a principal sound a fifth (musical fifth) higher than the original, and the one-third part has a sound an octave higher than the two-thirds; thus becoming, in fact, the same as two new strings.

Third: If the point of contact is moved to one eighth the length of the string, which is the center between the nearest nodes, the whole string is instantly silent and dead. At that point, *and no other on the string*, the harmonies all cease *with* the principal, when touched sufficiently to stop vibration at that point; showing (I think, quite conclusively) that the four divisions above stated are *all* that the string naturally has.

Fourth: A still further proof of this is seen in the fact that the point at one-eighth the length from the end or bearing of a string is the proper point for concussion to produce the loudest and most powerful sound of which the string is capable, as that not only preserves the principal vibrations it assumes intact, but contact at that point is also an instantaneous aid to the string in making the subdivisions of vibrations which it seems compelled by the law of equilibrium to make. There are, however, some variations from this rule, which, from a superficial view, might appear quite practical, viz: the well-known fact that the point for concussion in piano-fortes, in the highest octave, is generally found to improve the tone by moving it nearer the bearing, say to about one-tenth or even one-twelfth the length of string. But this simply proves that the bearing place itself is too vibratory, and hence the necessity of moving the striking place for the hammer sufficiently near to answer the *additional* vibrations of the resting place.

There may be also some variations in the lowest octave of the instrument. But as that part is so far from any rule, by making the strings so much shorter in proportion to the other parts of the scale, and by loading them with covering to suit the notions of any ignorant manufacturer of that instrument, who may imagine that the laws of sound should conform to his particular ideas and benefits, and various imperfect modes of making and bracing the sounding-boards, connecting the bridges, &c., all of which have a direct effect upon the action and freedom of the string to give a full sound, it would be quite as useless as it is impossible to apply any law or rule to it.

DOES ALL SOUND MOVE AT THE SAME VELOCITY?

We extract the following from the London *Photographic News*:—At this season of violent thunderstorms, our readers will be interested to know of some observations which have recently been made respecting the phenomenon of thunder. It has generally been considered that sound moves at a uniform velocity of 1,142 feet per second; and in every book on the subject rules are given by which the distance of any source of sound, such as a firearm or a flash of lightning, may be ascertained by estimating the number of seconds and fractions of a second which elapse between the occularly-observed time of the occurrenee of the phenomenon and the hearing of the sound which accompanies it. Doubtless many persons have in this manner amused themselves by estimating the distance off which one of of the recent violent lightning flashes has been, and have taken comfort from the idea that, if a certain number of seconds have elapsed *after* the flash has taken place before the thunder is heard, they are safe from its effects; falling into the very common error of mistaking the cause for the effect. The Rev. S. Earnshaw has, however, been engaged in some extremely interesting mathematical investigations respecting the phenomenon of sound, and has arrived at the theoretical conclusion that violent sounds are propagated far more rapidly than gentle sounds, and that therefore all reasoning upon the distance of the flash, based upon the lapse of time between it and the thunder, is fallacious. Many instances of this fact are adduced in corroboration of the theory, in which the clap of thunder followed immediately after the lightning, when, judging from the distance which the latter was from the observer, there should have been an interval of many seconds duration. These and similar instances have induced the above-named gentleman to enter upon a mathematical investigation of the theory of sound, and he arrives at the conclusion, contrary to the hitherto universally received opinion, that there is no limit to the velocity with which a violent sound is transmissible through the atmosphere, provided the phenomenon which produces the sound be sufficiently violent. Hence, it is probable that there is no sound which is propagated faster than a clap of thunder, its genesis being especially violent. This theory seems also capable of explaining the rumbling, rolling noise of thunder. It is only necessary to imagine that the sound at its origin is broken up, either by partial interruption or reflection, into several sounds of different degrees of violence. They would thus be propagated with different degrees of rapidity, and would therefore not fall upon the ear, if it were at any distance off, with a sudden crash, but in a series of minor claps, or as a rattle. If this theory be true, the report of a cannon should travel faster than the human voice, and that of thunder faster than either. This, we think, could easily be put to a *crucial* test.

OLIVER EVANS AND THE FORM OF SHIPS

There are some men who live in advance of their age, and whose merits are never appreciated until they are numbered with the departed prophets. Of this class we have a brilliant example in Oliver Evans, of Philadelphia. In our opinion, he was the first inventor of the locomotive steam engine, and he actually constructed one which carried a load through the streets of Philadelphia in 1784-5. Another interesting fact in relation to his discoveries respecting the best form for ships has just been published by the *American and Gazette*, of Philadelphia, the editor of which states that he has lately examined drawings and specifications for a boat that was to be built for a son of Mr. Evans, in 1813, at Pittsburgh, Pa. It is stated that these diagrams represent a boat with the same long, sharp entrance, and fine lines at the stern as the fastest boats now running on Lake Erie. Oliver Evans' propositions and reasoning on the lines, proportions, resistances, and speed of boats were as follows:—

"The velocity may be increased one, two, three, or any number of times with no greater displacement of water, by simply elongating the bow the same number of times the ship's breadth of beam, and thus reducing or removing the direct resistance of the water forward of the ship. This direct resistance is, with a bow having an angle of 45 degrees, theoretically the same as to a square end or bow, because a mass of water corresponding in shape would, in that case, be driven forward of the boat. If the bow is carried forward six or ten times this half diameter of the ship, the calculation for resistance, at the bow alone, rises to high proportions. It is thirty-six times as great with the first-named form as with one six times as long, if the speed is increased six times also; and it is one hundred times as great with the square bow as with one ten times as long, the speed being increased ten times." After going through the demonstration that resistance is increased as the cube of the velocity of a moving steamship, he says:—"Because water opposes resistance as the cube of the velocity with which it is moved in any given time, therefore, to make a boat to run fast with little force we must construct it so that it will *move the water slowly while it runs fast.* Whereas, water is a heavy substance of great inertia, it cannot be put instantly in rapid motion, the sides of a bow ought to be a serpentine line, to make it part the water very easily at first, but faster until it comes near the middle of a bow, and then slower until it comes to the greatest width of the boat. And the stem must be as sharp as the bow and of the same taper and shape, to let the water close slowly at first, but faster near the middle of the taper and slower at the last, so as to leave no eddy or wake behind her. If these calculations be well-founded, it is impossible to make boats too sharp at the bow and stern to be driven by steam engines; therefore, if it be not too late, make the bow and stern of my boat very sharp, for it appears that the greater the velocity that we wish to run, the greater advantage in having sharp bow and stern. It appears that for every time we add the width of the beam to the length of bow, we can add one more velocity without moving the water sideways any faster, and if the bow be 2, 3, 4, 5, or 6 times in length of beam, we can run with a velocity 2, 3, 4, 5 or 6 times as fast as with the bow once the length of the beam, and move the water sideways no faster. This is a subject worthy of consideration."